Entropic cosmology: a unified model of inflation and late-time acceleration

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Holography is expected as one of the promising descriptions of quantum general relativity. We present a model for a cosmological system involving two holographic screens and find that their equilibrium exactly yields a standard Friedmann-Robertson-Walker universe. We discuss its cosmological implications by taking into account higher order quantum corrections and quantum nature of horizon evaporation. We will show that this model could give rise to a holographic inflation at high energy scales and realize a late-time acceleration in a unified approach. We test our model from the SN Ia observations and find it can give a nice fit to the data.

PACS numbers:

I. INTRODUCTION

Einstein's classical general relativity is commonly acknowledged as the theory of gravitational interactions for distance sufficiently large compared to the Planck length. This validity has become the foundation of modern cosmology in describing the dynamics of our universe. However, its quantum effects are expected to become important at high energy scale, namely at very early time of cosmological evolution. Especially, the quantization of Einstein gravity has long been known to be perturbatively nonrenormalizable. Various attempts on solving this issue have been intensively studied in the literature. It is widely believed that the quantization of Einstein gravity is related to the solution to Big Bang singularity of our universe.

As early as the discovery of black hole thermodynamics by Bekenstein[1] and Hawking[2], people have realized that a nonperturbative feature of Einstein gravity may be related to the holographic thermodynamics. Especially, 't Hooft proposed the holographic principle as a particular property of quantum gravity which states that the description of a volume of space can be thought of as encoded on a boundary of this system, preferably a light-like boundary like a gravitational horizon[3]. Subsequently, this issue is extensively discussed in cosmology[4, 5] and recently realized in the context of developments in string theory[6]. Therefore, it provides a promising description of quantum general relativity. An extended holographic picture was conjectured by Verlinde and in this scenario Einstein gravity is originated from an entropic force arising from the thermodynamics on a holographic screen[7](see also [8–10] and references therein for earlier studies along this direction). In this scenario, however, there exists a key controversial issue whether gravity is fundamental or emergent[11, 12], and thus relevant reinterpretations of Verlinde's Entropic force was discussed in [13].

Recently, a much explicit formulation of Entropic gravity theory was suggested by Easson, Frampton and Smoot (EFS) , in which the general relativity is still a fundamental theory but including a boundary term. In this picture the holographic entropic force arises from the contribution of boundary terms[14]. This model was soon applied to realize current acceleration [14] and inflationary period at early universe [15](see [16] for a study of entropic inflation within Verlinde's proposal). In both Verlinde's proposal and EFS one, one should be very careful of whether they can explain the current cosmological observations consistently[17]. First of all, let us assume gravity is entropic, when applied into cosmology, we should explain why our CMB radiation and the holographic screen are not in thermal equilibrium. This problem is very manifest, since the temperature of our CMB radiation and thus our universe is observed as $T_{CMB} = 2.73$K, but the horizon temperature can be easily estimated as $T_H \sim O(10^{-30})$K. In such a thermal system out of equilibrium, the heat transfer ought to be very strong. Therefore, if such a heat transfer occurs today and leads to an acceleration, then our universe is always accelerating in the past without radiation and matter dominations since the temperature gap is always very large at early times (we have $T_H \sim T_{CMB}^2 \sqrt{G}$ and therefore this system can only be in thermal equilibrium at Planck scale). This conclusion explains why the modified Friedmann equation appeared in the EFS paper is so different from the normal one in Einstein gravity.

Concerning the above question, in the present work we are interested in how to recover a standard Friedmann equation from an entropic cosmological system. We suggest that there exist two holographic screens with one being the approximate Hubble horizon which is similar to the de-Sitter (dS) horizon, while the other Schwarzschild horizon. Each screen has its own thermodynamics which is independent of that of the other. Consider the classical dynamics of such a cosmological system, we find that its thermal equilibrium corresponds to a standard FRW universe. Therefore this model is able to explain the normal thermal history observed in our universe. Moreover, we consider quantum corrections to the area entropy and

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obtain an EFS universe a little bit deviating from thermal equilibrium. We study the cosmological implications of this model, and find that it can drive an entropic expansion at early universe and realize the late-time acceleration in a unified approach.

The letter is organized as follows. In section II we briefly review the idea of entropic force from the viewpoint of an effective action description. In section III we provide an explicit formulation of the FRW universe from the classical thermal equilibrium state of double holographic screens. Effects from higher order quantum corrections of the holographic entropy and a quantum evaporation of the inner horizon are studied in this model. In section IV we study the cosmological implications of this model involving quantum corrections. Our results show that in this model a holographic inflation could be obtained at high energy scales, and a quantum evaporation process of the inner horizon is related to the realization of the late-time acceleration. We confront this model with the latest observations at the end of this section, and the results are in agreement with observational constraints. Section V presents a summary and discussions of the related works. We take the convention $c = k_B = \hbar = 1$ in this letter.

II. REVIEW OF ENTROPIC FORCE AND EFFECTIVE ACTION DESCRIPTION

We start with a discussion on the standard approach to studying quantum field theory. The viewpoint of modern physics suggests any fundamental theories can be described by an effective action at certain energy scale. So does Einstein gravity. Under this assumption, we can obtain the equations of motion to describe the dynamics of these theories from a variational principle. In a usual quantum field theory, we can integrate out the boundary terms which do not change its physics in Minkowski spacetime. When dealing with a gravitational system, however, one should be very careful of the boundary terms which may play an important role of the background evolution.

In general, the effective action of a gravitational system including matter fields and surface terms is described by

$$\mathcal{I} = \int_M \left(\frac{R}{16\pi G} + \mathcal{L}_m \right) + \oint_{\partial M} \mathcal{L}_b , \qquad (1)$$

where R is the Ricci scalar of the whole spacetime, $\mathcal{L}_m$ is the Lagrangian of matter fields living in the bulk, and $\mathcal{L}_b$ is the corresponding Lagrangian describing the physics of the boundary. Clues from string theory and AdS/CFT indicate that the boundary terms should include the extrinsic curvature of the boundary and holographic dual gauge theories. In this Letter, we take this action as our starting point to describe a holographic picture of the evolution of our universe.

By varying the action with respect to the metric, we can obtain the Einstein field equation as follows,

$$R^{\mu\nu} - \frac{1}{2}Rg^{\mu\nu} = 8\pi G T_m^{\mu\nu} + J_b^{\mu\nu} , \qquad (2)$$

in which the last term J_b is a current describing the exchange of energy and momentum between the bulk and the boundary. In usual, it is described by a delta function in order to satisfy the locality of the theory. However, in the frame of a holographic picture, this term is determined by the holographic description of boundary physics and so is a nonlocal effect, which in our letter corresponds to an entropic force in the universe[1].

Now we assume the boundary physics can be described by thermodynamics satisfying a holographic distribution. Therefore, the number of degrees of freedom on this holographic screen is proportional to its area which takes $N \propto A$. In this case, the classical holographic entropy on this screen is given by

$$S_b = \frac{A}{4G} = \frac{\pi}{G}r_b^2 , \qquad (3)$$

where r_b is the radius location of the boundary surface. The entropic force is determined by the variation of energy with respect to the radius,

$$F_e = -(\frac{dE}{dr})_b = -(T\frac{dS}{dr})_b = -\frac{2\pi}{G}T_b r_b , \qquad (4)$$

in which T_b is the temperature of the boundary system and we have applied Eq. (3) in the above formula. According to the Unruh effect, when a test particle with mass m is located near by the horizon, the variation of the entropy on this horizon with respect to the radius takes the form of

$$\frac{dS}{dr} = -2\pi m , \qquad (5)$$

and thus the combination of Eqs. (4) and (5) yields an entropic acceleration a_e as follows,

$$a_e \equiv \frac{F_e}{m} = 2\pi T_b . \qquad (6)$$

The corresponding entropic pressure takes a negative form $P_e = F_e/A_b = -T_b/2Gr_b$, and so is expected to drive the current acceleration of our universe.

Now we apply the above results into a homogeneous and isotropic flat Friedmann-Robertson-Walker (FRW) universe described by

$$ds^2 = dt^2 - a(t)^2 dx^i dx^i , \qquad (7)$$

and the Einstein field equation gives the acceleration equation for the scale factor as follows,

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3}(\rho + 3p) + \frac{a_e}{L_b} , \qquad (8)$$

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where L_b is a length scale relevant to the location of the holographic screen, of which a natural choice is to near by the Hubble horizon[2]

$$L_b = r_H = \frac{1}{\beta H} \; , \qquad (9)$$

with the horizon temperature being

$$T_H = \frac{\beta H}{2\pi} \; , \qquad (10)$$

and $H \equiv \dot{a}/a$ is the Hubble parameter of the universe. Finally, we arrive at a modified Friedmann acceleration equation as follows,

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3}(\rho + 3p) + \beta^2 H^2 \; . \qquad (11)$$

From this equation we can learn that, if the coefficient β^2 is of order $O(1)$, the Friedmann equations describing the evolution of our universe would be changed too much by taking into account such a holographic screen, which is hardly able to recover the usual form in radiation and matter dominated periods. This point also appeared in Ref. [14], and the authors of that paper suggest to modify the coefficients before the temperature and introduce an alternative entropic acceleration.

III. FORMULATING FRW UNIVERSE FROM THERMAL EQUILIBRIUM OF DOUBLE HOLOGRAPHIC SCREENS

The key reason leading to the above over-modified Friedmann equation is that in this gravitational system we have only considered a single holographic screen related to the Hubble horizon. In this case, there exists a temperature gap between the bulk universe (CMB temperature) and the horizon, which indicates that the whole gravitational system is not in thermal equilibrium and all the particles in the bulk universe will fall down onto the horizon due to such an unbalanced effect.

A. A delicate picture of double holographic screens

In such a cosmological system, one another holographic screen ought to be taken into account, i.e., the Schwarzschild horizon. This horizon is motivated by physics of black holes, which is another promising approach to understanding the quantum information of Einstein's gravity. A black hole has many remarkable properties, namely, the association with thermodynamics

and the reflection of holography introduced at the beginning of the current Letter. The simplest black hole solution in four dimensions corresponds to a Schwarzschild spacetime discovered many years ago. In this solution, the whole geometry is divided into two causal independent regions by an event horizon at the radius

$$r_S = 2GM = 2G \int_M \rho dV = \frac{8\pi G\rho}{3\beta^3 H^3} \; . \qquad (12)$$

Its corresponding temperature is given by

$$T_S = \frac{1}{8\pi GM} = \frac{3\beta^3 H^3}{32\pi^2 G\rho} \; , \qquad (13)$$

and therefore, according to the relationship obtained in Eq. (6), we obtain a modified entropic acceleration

$$a_e = 2\pi(T_H - T_S) = \beta H \left(1 - \frac{3\beta^2 H^2}{16\pi G\rho} \right) \; , \qquad (14)$$

which reflects a competition of entropic effects from the outer horizon and the inner horizon. Interestingly, if these two holographic screens are delicately on thermal equilibrium with $T_H = T_S$ and we choose the coefficient $\beta = \sqrt{2}$, one can recover the exact form of the traditional Friedmann equation

$$H^2 = \frac{8\pi G}{3}\rho \bigg|_{\text{equilibrium}} \; , \qquad (15)$$

which coincides with the normal Friedmann equation. In this case one can check the Schwarzschild radius is smaller than the Hubble radius. Moreover, the continuous equation for matter fields in the bulk spacetime satisfies

$$\dot{\rho} + 3H(\rho + p) = 0 \; , \qquad (16)$$

when there is no coupling to the boundary surfaces. These two equations of motion are self-complete and yields the normal evolution of our universe.

In the following we depict this holographic picture with a cartoon sketch as shown in Fig. 1. In this figure, one can see that any matter fields in the bulk spacetime would feel two different entropic forces of which the directions are opposite. Therefore, the energy stored in the bulk spacetime will transfer to the holographic screen with lower temperature until two temperatures equal to each other. Once $T_H = T_S$, the universe will arrive at a delicate balanced state and evolves according to its own equations of motion without energy exchanges among these screens. Additionally, during the whole evolution of the universe the second law of thermodynamics is preserved.

B. A modified Friedmann equation with quantum corrections

Up to now, we have studied the holographic description of an FRW universe with double screens in classical
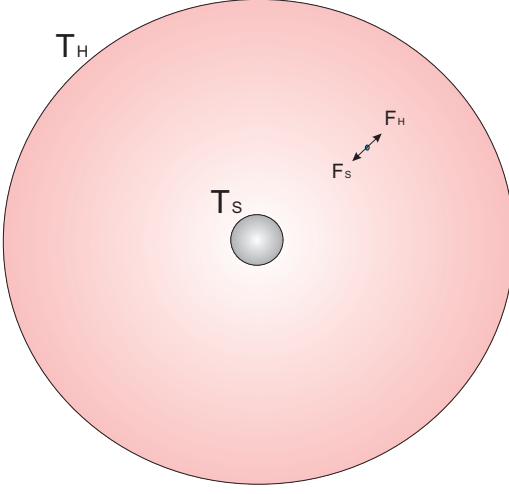
FIG. 1: A sketch of the dynamics of a test particle under double holographic screens. The inner screen is the Schwarzschild horizon and the outer one is comparable to the Hubble horizon. The particle will feel two opposite entropic forces and finally fall into a delicate balanced state when two screens are on thermal equilibrium.

level. However, it has been studied for many years in the field of quantum Einstein gravity and string theory that, the form of a holographic entropy should be improved when higher order quantum corrections are taken into account[15]. Namely, both the number of string states[18–20] and the holographic renormalization group flow[21] yields an improved form of the entropy with leading order corrections as follows,

$$S_i = \frac{1}{4G}\left(A_i + g_i G \ln \frac{A_i}{G} + ...\right) , \qquad (17)$$

where $i = S, H$ denotes the inner and outer horizon respectively. In the above formula, the coefficient g_i is determined by specific environment and here we would like to leave it as a free parameter.

In this case, the entropy force of a holographic screen is changed to be

$$F_e = -T\frac{dS}{dr} = -\frac{2\pi}{G}rT\left(1 + \frac{gG}{4\pi r^2} + ...\right) , \qquad (18)$$

at leading order. With double holographic screens, we obtain an improved acceleration equation, which is expressed as

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3}(\rho + 3p) + f(\rho, H) , \qquad (19)$$

with the form of surface function being

$$\begin{aligned} f(\rho, H) &= \beta^2 H^2\left(1 - \frac{3\beta^2 H^2}{16\pi G\rho}\right) \\ &+ \frac{g_H G\beta^4 H^4}{4\pi}(1 - \frac{27g_S\beta^6 H^6}{1024g_H\pi^3 G^3\rho^3}) + ... \end{aligned} \qquad (20)$$

C. Dissipating equation of the Schwarzschild horizon

In the physics of quantum gravity, there is an important lesson from holography, i.e., when there exists Hawking radiation, the boundary horizon loses a small amount of its energy[2]. This can be achieved similar to the well-known studies of a massive primordial black hole[22] (see the recent study [23–26]). In a generic cosmological background, its dissipating equation is given by[25],

$$\frac{dM}{dt} = -\frac{\alpha}{M^2} + \kappa\frac{M^2}{t^2} . \qquad (21)$$

Here the α term describes the evolution of the Schwarzschild horizon via Hawking radiation, and so $\alpha = 1/15360\pi G^2$. Moveover, the κ term denotes the accretion process and the value of κ is determined by the background cosmological evolution at initial moment, which in a usual case corresponds to a critical point t_C when the whole system is on the thermal equilibrium and the standard Friedmann equation is recovered. Thus one gets $\kappa = 16\rho_C t_C^2$.

The dissipating equation (21) can not be solved in an analytic approach. However, one may notice that, for such a horizon evolving at late time of the universe, the κ term is strongly suppressed by the age of our universe. As a consequence, one can solve the dissipating equation and obtain an evaporation time of the form

$$\tau_S \simeq \left(\frac{M_C}{10^{15}g}\right)^3 \times 10^{17}s , \qquad (22)$$

where M_C denotes the Schwarzschild mass at the critical moment. From this relation, we find that in order to let the age of such a Schwarzschild horizon in the order of our universe, there has to be $M_C \sim O(10^{38}\text{GeV})$ at the critical moment. We leave the explicit values of the coefficients to be determined or constrained by astronomical observations which will be discussed later. Note that if this horizon has not evaporated out completely until today, in principle it could be expected to be observable by CMB experiments. Namely, the radiation arising from the evaporation of the inner horizon could bring an amplification of CMB spectrum at sub-Hubble scales. This issue deserves a study in detail in the future[27].

In addition, we would like to comment on the quantum decay of the outer horizon. Since this horizon is similar to the event horizon of the dS spacetime, according to the work of Coleman and De Luccia[28], the decay time of a meta-stable dS vacuum has an approximate expression $\tau_H \sim \mathcal{P}_H^{-1}$. By neglecting all the sub-exponential factors, we have

$$\mathcal{P}_H \sim e^{-\pi/GH^2} , \qquad (23)$$

which is exponentially suppressed by the age of the universe. Therefore, we can neglect the radiation emission of the outside horizon throughout the whole cosmological evolution.

IV. COSMOLOGICAL IMPLICATIONS

We now study the cosmological implications of double holographic screens. Specifically, we discuss realizations of the late-time acceleration and entropic evolution of early time universe respectively. We find there might be a potential way to unify these two processes together in the frame of entropic cosmology.

A. Early time evolution of the holographic universe

According to the well-known knowledge of thermodynamics, a system usually evolves from an unstable state to the equilibrium. This process is described by a generalized acceleration equation appeared in Eq. (19), but we still need one another equation of motion to solve this system self-complete. We consider the conservation of the whole energy in this system, which sums up the contributions from the double holographic horizons and the universe, which can be described by[3]

$$\Delta E_u + \Delta E_S + \Delta E_H = -p\Delta V , \qquad (24)$$

where $\Delta E_i = T\Delta S_i$ for $i = S, H$ respectively. This yields an improved continuous equation

$$\dot{\rho} + 3H(\rho + p) = \Gamma , \qquad (25)$$

with an effective coupling term Γ being

$$\Gamma = \frac{27\beta^6 H^6}{1024\pi^3 G^3 \rho^3}\dot{\rho} + \frac{3\beta^2 H\dot{H}}{4\pi G}\left(1 - \frac{27\beta^4 H^4}{256\pi^2 G^2 \rho^2}\right) , \quad (26)$$

in classical level. One can check when $\beta = \sqrt{2}$ and $T_H = T_S$, the coupling Γ vanishes and Eq. (25) is in agreement with the normal continuous equation (16).

Consider a radiation dominated universe at high energy scales. We assume the equation of state for the radiation in the universe is determined by its intrinsic physical nature which gives $p = \rho/3$. To combine the equations of motion (19) and (25) and only keep the leading order quantum correction to the entropy, one can approximately solve out the following solution for the Hubble parameter

$$H^2 = \frac{8\pi G}{3}\left[\rho + \frac{8(g_H - 4g_S)}{69}G^2\rho^2 + ...\right] , \qquad (27)$$

at early universe. During the semi-analytical derivation, we find again that β is required to be $\sqrt{2}$ approximately in order to make sure the above calculation self-consistent, which indicates the system would approach to

the thermal equilibrium state along with the cosmological evolution.

From Eq. (27), one can see that a standard Friedmann equation can be achieved if $g_H = 4g_S$ even without a thermal equilibrium at early times. There exist two branches of cosmological solutions at high energy scales. The first one describes an expanding universe with $g_H > 4g_S$, and the Hubble parameter is proportional to the energy density at high energy scales. In this branch the ρ^2 term could make the early time inflation much easier be realized, so would provide an implement of holographic inflation as studied in [15, 16]. In the following we will illustrate its possibility in the example of a radiation dominated universe.

In the case of $g_H \neq 4g_S$, the Hubble parameter will be dominated by the second term in the rhs of Eq. (27) when the energy density of the universe reaches a critical value

$$\rho_C \simeq \frac{69}{8(g_H - 4g_S)G^2} , \qquad (28)$$

for a radiation dominated entropic universe. One can solve the equations of motion and find that the energy density of the universe evolves as

$$\rho \simeq \sqrt{\rho_C^2 - \frac{512\pi^3 t}{27G^{9/2}(g_H - 4g_S)^{5/2}}} , \qquad (29)$$

where we set the initial moment of the entropic inflation $t_i \to -\infty$ and choose $t_C = 0$ corresponds to the beginning moment of normal Friedmann equation. To proceed, we obtain the form of Hubble parameter as follows,

$$H(t) \simeq 24.25 \times \frac{(-t)^{1/2}}{[G(g_H - 4g_S)]^{3/4}} , \qquad (30)$$

when $|t| \gg 1$. In order to characterize the inflationary process, it is convenient to introduce the slow-roll parameter ϵ, which is in form of

$$\epsilon \equiv -\frac{\dot{H}}{H^2} \simeq 2.06 \times 10^{-2}\frac{[G(g_H - 4g_S)]^{3/4}}{(-t)^{3/2}} . \qquad (31)$$

This parameter can be much less than unity when $|t| \gg \sqrt{G(g_H - 4g_S)}$. Consequently, we obtain a period of holographic inflation.

Note that, if we assume the critical scale obtained above corresponds to the critical mass appeared in Eq. (22), it requires ρ_C to be around or less than $O(10^{-3}M_p)^4$ with $M_p \equiv 1/\sqrt{G}$. Therefore, in order to let the inner horizon evaporate within the age of our universe, one expects that the value of $|g_H - 4g_S|$ should be finely tuned as large as $O(10^{12})$. As an explicit example, we choose $|g_H - 4g_S| = 10^{16}$ and thus get the efolding number

$$\mathcal{N} \equiv \int^t H(t)dt \simeq 1.62 \times 10^{-11}(-M_p t)^{3/2} . \qquad (32)$$

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[3] We notice our conservation equation is manifestly different from Eq. (4) in [17] and consequently the second Friedmann equation is no longer in form of $\dot{H} = -4\pi G(\rho + p)$.

where we introduce the Planck time $t_p = \sqrt{G}$.

We plot the evolution of the Hubble parameter H and the slow-roll parameter ϵ as functions of the efolding number $\mathcal{N}$ in Figs. 2 and 3. From these figures, one can read that for $\mathcal{N} = 60$ there is $\epsilon \simeq 3 \times 10^{-5}$ and $H = 3.8 \times 10^{-7} M_p$. Finally, we find that in the model of double holographic screens the universe could experience an exponential expansion at early times due to the effect of entropic force.
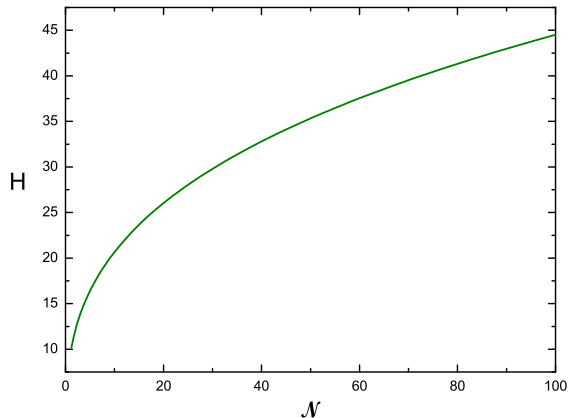
FIG. 2: A plot of the evolution of the Hubble parameter H in the scenario of holographic inflation as a function of the efolding number $\mathcal{N}$. In the numerical computation, we choose $g_S = 0$ and $g_H = 10^{16}$. The Hubble parameter is in unit of $10^{-8} M_p$.
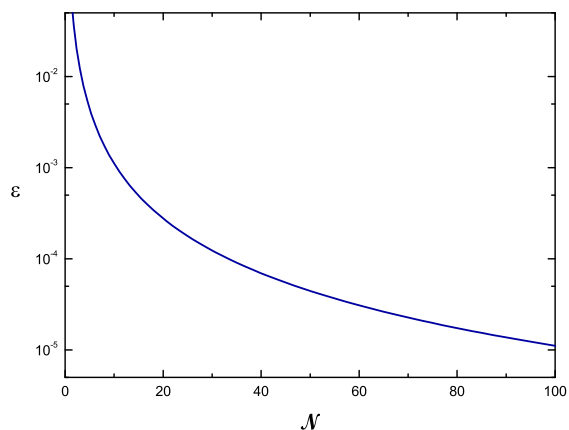

FIG. 3: A plot of the evolution of the slow-roll parameter ϵ in the scenario of holographic inflation as a function of the efolding number $\mathcal{N}$. In the numerical computation, we choose $g_S = 0$ and $g_H = 10^{16}$.

The second branch corresponds to the case of $g_H < 4g_S$. In this case we expect there exists a cosmological bouncing solution which may avoid the initial singularity. Since this topic is beyond the aim of the current Letter, we would like to leave it in future studies.

B. Late-time acceleration and comparing with the SN Ia data

Along with the decreasing energy scale, the universe will gracefully exit from a primordial epoch either a normal expansion or a holographic inflation, and then smoothly link with the standard thermal history as we observed. During this period, our universe has already arrived at the delicate thermal balanced state with its evolution satisfying the standard Friedmann equation. However, as mentioned in the above section, one should take into account the horizon evaporation via Hawking radiation. Therefore, the double holographic screens can also lead to the late-time acceleration of the universe once the inner horizon evaporates.

The late-time acceleration can be realized via the equation of motion (19) where the form of f approximately takes $\beta^2 H^2$ which coincides with the result obtained in Ref. [14]. However, we leave the coefficient β to be a free parameter due to lacks of detailed information of the inner horizon evaporation. In principle, we can constrain the model by fitting to the Type Ia Supernovae (SN Ia) data and give the constraints on the model parameter β. In order to see the rationality of the model transparently, we plot the distance moduli given by our model with specific values of β in Fig. 4 as well as the SN Ia data given by the "union" compilation[29]. We use the metric theory of gravity and the general formula of the luminosity distance is given by:

$$d_L(z) = (1+z) \int_0^z \frac{dz'}{H(z')}, \qquad (33)$$

where z is the redshift define by $a_0/a = 1 + z$. In the ΛCDM model, the Luminosity distance takes the form[30]:

$$
\begin{aligned}
d_L = {} & c(1+z)H_0^{-1}|\Omega_k|^{-1/2}\mathrm{sinn}\{|\Omega_k|^{1/2} \\
& \times \int_0^z dz[(1+z)^2(1+\Omega_M z) - z(2+z)\Omega_\Lambda]^{-1/2}\} ,
\end{aligned}
$$
$$(34)$$

where $\Omega_k = 1 - \Omega_M - \Omega_\Lambda$, and "sinn" is sinh for $\Omega_k > 0$ and sin for $\Omega_k < 0$. The blue dashed line and the green line are given by $\beta^2 = 1$, 2 respectively, and the red solid line is given by the ΛCDM model. From the plot, we can find a nice fit of the entropic acceleration to the SN Ia data for the distance moduli given by our model are well consistent with the data points at low redshift. The idealistic case with $\beta^2 = 2$ deviates from the observation at high redshift regime. This signature is

understandable since at high redshift regime the universe still satisfies the standard Friedmann equation and the double holographic screens are on thermal equilibrium as well. A more explicit investigation on this issue should be addressed in future in combination of the detailed study on dynamics of horizon evaporations.
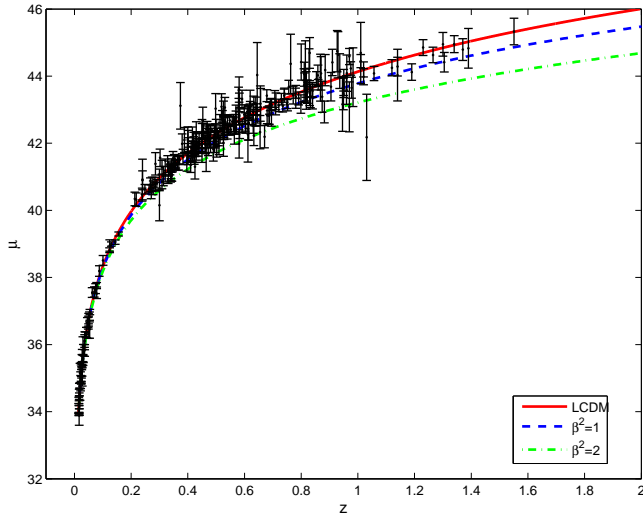

FIG. 4: A numerical plot of the Hubble diagram caused of entropic acceleration and its comparison to ΛCDM. The red solid line is given by ΛCDM model, the blue dashed line is given by $\beta^2 = 1$ and the green line is from $\beta^2 = 2$. The black dots with the error bars are the SN Ia "union" compilation sample.

V. CONCLUSION AND DISCUSSION

Since awaken by Verlinde, the idea of entropic force has become an important issue and its phenomenological applications were soon considered in an FRW universe[31–37], and is expected to explain the current acceleration of our universe[38], and drive an inflationary period at early times[16], and its effects on spherical symmetric spacetime were discussed in [39–47], and see [48–57] for relevant discussions. The frontier of modern physics suddenly goes back to pre-Einstein time one hundred years ago.

In this Letter, we extended the picture of entropic cosmology and suggested a scenario of double holographic screens to explain the past thermal expansion of our universe. We also studied the quantum signatures of this model motivated from physics of quantum gravity, and specifically we considered the higher order quantum corrections to the holographic entropy and the process of horizon evaporations. We found that the higher order quantum corrections to the entropic force may give rise to an implement of holographic inflation. In the meanwhile, the evaporation of the inner horizon could bring a realization of late-time acceleration. In order to let

this acceleration happen at current time, we found there exists a fine-tuning to the coefficients of higher order corrections. We test our model from the SN Ia observations, and find it can give a nice fit to the SN Ia data. The unification of inflation and dark energy era was earlier discussed in Refs. [58, 59] by introducing a phantom degree of freedom.

We would like to point out that the model we considered is still a toy model with many detailed clues ignored. Among them the most important issue is the study of primordial perturbations seeded by statistic fluctuations on the holographic screens, since we expect these thermal fluctuations could give rise to a nearly scale-invariant spectrum so that explain the CMB observations. We will perform a much complete and careful study on this issue in near future.

At the end of this Letter, we would like to make a few comments on the possibility of the avoidance of big bang singularity in entropic cosmology. In the main text, we have studied the cosmological implications of the model of double holographic screens at early universe by considering the higher order quantum corrections. Moreover, in quantum physics, there could be more arguments supporting the avoidance of the Big Bang singularity. Namely, as the Heisenberg uncertainty principle states, in quantum theory a test particle is described by a wave packet, which moves in the bulk spacetime. Consider the measurement of the absolute position of this particle. It could be anywhere since the particle's wave packet has non-zero amplitude, meaning the position is uncertain. The Heisenberg uncertainty principle requires, $\delta r \delta E \geq \frac{1}{2}$. To combine the above uncertainty relation and Eq. (4), one can obtain a minimal length scale for the thermal system

$$\Delta r \simeq \sqrt{\frac{G}{4\pi Tr}} \sim l_{pl} \ , \tag{35}$$

which implies that our universe cannot be shrunk into trans-Planckian scale. Therefore, we may get a nonsingular cosmic evolution of the universe at early times. In this case, we expect the model could realize the late-time acceleration and also avoid the initial big bang singularity in a unified approach without quantum instability[4]. If this model has a matter dominated contraction, it was found that both the thermal [71] and quantum [72–75] fluctuations are able to provide a scale-invariant spectrum with local featured signatures[76, 77] and sizable non-Gaussianities[71, 78], which may be responsible for the current cosmological observations. We note the study

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[4] It is widely noticed that in the frame of standard Einstein gravity, a nonsingular bounce model offer suffers from quantum instability due to a ghost degree of freedom[60, 61], this statement can also be extended into the cyclic cosmology[62–65]. Recently, a nonsingular bounce model was achieved in the frame of nonrelativistic gravity theory[66–68](see also [69, 70]).

of entropic force with Heisenberg uncertainty principle in the original Verlinde's model appeared in [79], and its extended form was analyzed in [80–85].

Acknowledgements

We wish to thank Robert Brandenberger, Damien Easson, Xian Gao, Miao Li, Stone Pi, Yun-Song Piao, Tao-

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